Emergence of Physics from a Singular Mathematical Core: Quantum Dynamics in Frustums at the Planck Scale

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This paper explores the transition from pure mathematics to physical reality, focusing on a central mathematical singularity where abstract symmetries and algebraic structures exist without physical form. As we move outward from this singularity, at a critical radius near the Planck scale, the first physical structures—discrete regions called frustums—begin to emerge. These frustums represent quantized regions of space where physical laws, such as those governing quantum mechanics, first come into existence. Within each frustum, quantum phenomena such as tunneling, superposition, and entanglement are key drivers of particle behavior, marking the transition from abstract mathematical principles to the familiar physical laws that govern spacetime, matter, and energy. This model suggests that quantum mechanics is not just a framework for understanding the microscopic world but plays a central role in the emergence of physical laws. We explore how tunneling between frustums, superposition of quantum states, and entanglement across space could form the foundation of physics as we know it.

Keywords: mathematical singularity, Planck scale, frustums, quantum mechanics, tunneling, superposition, entanglement, quantum dynamics, emergence of physics, spacetime quantization, cosmology, early universe. 39 pages.

1. Introduction

Motivation

In many theories of physics, there is a continuous quest to understand the transition from pure mathematical constructs to the physical universe as we observe it. This paper explores the possibility of a central singularity, representing the origin point of pure mathematical laws, where no physical properties yet exist—neither space, time, nor matter. At this singularity, mathematics governs completely, with symmetries, algebraic structures, and geometric relationships forming the foundation. As one moves outward from this singularity, a transition occurs at a critical radius, hypothesized to be near or at the Planck scale, where physical phenomena begin to emerge.

The concept of a Planck scale boundary is not new. In quantum gravity and other high-energy physics theories, the Planck scale represents the smallest meaningful unit of measurement in spacetime, where quantum effects of gravity become significant, and classical descriptions of spacetime break down. At this scale, the fabric of spacetime is thought to exhibit granular or quantum characteristics, suggesting that the laws of physics as we know them begin to materialize from more fundamental, abstract principles.

In the model proposed here, we imagine that as we approach a certain radius from the singularity, the mathematical laws begin to give rise to spherical frustums, which represent discrete regions of space where physics begins to take form. These frustums are geometrical regions that originate from the central singularity, representing the first physical manifestations of space and matter. The shape and boundaries of these frustums are not arbitrary but governed by mathematical principles that are still present at the threshold between mathematics and physics.

This transition from a purely mathematical realm to one where physics takes over suggests a natural boundary at the Planck scale. The frustums, which emerge at this threshold, are the building blocks of the physical universe. It is in these regions that the fundamental quantum mechanical principles, such as tunneling, superposition, entanglement, and quantum interference, begin to manifest, influencing how particles behave within these discrete spatial cells. This offers a rich framework for exploring how quantum mechanical phenomena emerge at

small scales and how they might relate to the deeper mathematical structures underlying the universe.

Objective

The primary objective of this paper is to explore how quantum mechanics—and more specifically, phenomena such as quantum tunneling, superposition, entanglement, and interference—manifests within the context of these frustums that originate from a central mathematical singularity. By studying the behavior of particles within these frustums, we aim to gain insights into how the transition from mathematics to physics occurs, especially at the Planck scale.

We propose a framework where each frustum acts as a distinct region of space, a geometric vessel within which quantum particles are confined. These regions are connected in a manner analogous to the way mathematical principles tie together different parts of an abstract space. We explore how quantum mechanical processes, such as tunneling between adjacent frustums, arise from the underlying mathematics of the singularity and how entanglement between particles in different frustums might reflect the deep, inherent connections of the mathematical structure from which they originate.

By addressing the behavior of quantum systems in this model, we hope to:

1. Understand how quantum mechanics could naturally emerge from pure mathematical laws as we approach the Planck scale.
2. Explore the dynamics of quantum tunneling between these frustums and the resulting wavefunction behaviors.
3. Examine the role of superposition and entanglement in these discrete regions and how they might provide insights into the geometry of spacetime at small scales.
4. Investigate the quantum interference patterns that arise when particles interact within and across the frustums, providing clues to how physical laws begin to organize.

This paper contributes to the broader effort of understanding how the universe transitions from a realm dominated by mathematical abstraction at the quantum level to the observable physical laws that govern reality as we know it. By

situating our model at the intersection of quantum mechanics and mathematical theory, we aim to provide a novel framework for thinking about the origins of physics itself, starting from the smallest scales of reality.

2. Mathematical Core and the Singularity

Singularity at the Center

At the heart of this model lies a singularity, a central point where physical laws do not yet exist—only pure mathematics governs. This singularity is unlike any physical point we typically encounter in physics, where physical quantities like energy, mass, or even spacetime curvature dominate. Instead, this central singularity is defined entirely by abstract mathematical structures. Here, the rules that dictate reality are expressed in terms of fundamental symmetries, geometric forms, and algebraic relationships.

This singularity represents the ultimate abstraction, a realm where familiar physical concepts like position, velocity, or force have no meaning. Only the relationships between abstract quantities, like numbers, geometric angles, or algebraic operators, have any significance. In this sense, the singularity can be thought of as a mathematical seed from which all physical reality emerges. Key mathematical concepts that define this singularity include:

- Symmetry: The singularity is governed by mathematical symmetries—rotational, translational, or even more abstract symmetries like those found in group theory. These symmetries will later influence the geometry of space and the properties of matter, but in the singularity, they are purely abstract and unmanifested.
- Algebraic Structures: Here, we can imagine algebraic systems like matrices, tensors, and even more complex mathematical objects, such as elements of Lie groups or Clifford algebras, existing in their pure form. These mathematical tools will ultimately dictate how physical quantities like energy or momentum behave, but within the singularity, they are simply abstract rules waiting to give birth to physics.

- Geometry: Geometry in this context refers not to the geometry of space as we know it, but to an idealized, purely mathematical geometry—topological forms and spaces that exist without embedding into physical reality. The curvature, dimensionality, and connectivity of this "space" are mathematical objects, devoid of any physical interpretation until later in the model when physical space emerges.

In this pre-physical state, the singularity serves as the foundation from which everything we recognize as physical reality is constructed. It is akin to the primordial source in cosmological models, except it is not defined by physical parameters such as temperature or density but by pure mathematical abstraction. There is no time, no distance, and no material—just the raw potential encoded in the language of mathematics.

Mathematics to Physics Transition

As we move outward from this central singularity, we approach a critical threshold, potentially at or near the Planck scale, where pure mathematical rules begin to translate into the familiar laws of physics. This transition is not abrupt but occurs gradually as the radius from the singularity increases. As we cross this threshold, space, time, and matter begin to take form, and the abstract mathematics that once governed purely hypothetical relationships begins to manifest as physical phenomena.

1. Planck Scale as the Boundary:
    - The Planck scale serves as a natural boundary in many quantum gravity theories because it represents the smallest meaningful unit of spacetime. At distances smaller than the Planck length, the classical conception of space and time breaks down, and quantum fluctuations dominate. In this model, the Planck scale acts as a critical radius, marking the point where the mathematical rules governing the singularity give rise to physical reality.
    - As the radius from the singularity approaches the Planck length, quantum effects become increasingly important. This is the region where the granularity of spacetime first emerges, and it is here that we begin to see the building blocks of reality—quanta of space,

quanta of time, and elementary particles—all of which are governed by the same mathematical structures that existed at the singularity.

2. Emergence of Space, Time, and Matter:
    - Space: Initially, the mathematical structure of the singularity is without physical dimensionality. However, as we approach the Planck scale, space begins to emerge. The frustums that form from the singularity are the first hints of physical space. These frustums are regions of finite area that define where physical laws begin to take hold. The geometry of these frustums—whether pentagonal, hexagonal, or other shapes—is still governed by the underlying symmetries and mathematics of the singularity, but now they manifest in the form of spatial regions.
    - Time: Just as space emerges at this boundary, so does time. Time begins as an abstract concept in the singularity, defined by its relationship to symmetry (for example, time translation symmetry), but as we move outward, time becomes a measurable quantity. The frustums also define regions where time can be experienced as a linear, forward-moving entity. The interactions between particles in these regions introduce the first instances of time-dependent phenomena.
    - Matter and Energy: Matter, in this context, emerges as the first quantum particles that inhabit the frustums. At the Planck scale, these particles are not yet the familiar atoms or subatomic particles we know, but more fundamental quantum fields or entities like strings or loops. Their behavior is still largely governed by the mathematical symmetries of the singularity, but they now have physical properties—mass, charge, spin—that interact according to physical laws.
3. Frustums as Quantum Regions:
    - The emergence of frustums at the Planck scale marks the first instance of physical space divided into discrete regions. Each frustum represents a confined region of space where quantum mechanics

takes over. These frustums are geometric volumes that are curved and connected in a manner dictated by the underlying mathematical rules. Importantly, the particles within these frustums are quantum mechanical in nature, subject to the rules of tunneling, superposition, entanglement, and quantum interference.

- The geometry of these frustums arises from the symmetries of the singularity. The tiling pattern, with its pentagonal and hexagonal regions, is a direct manifestation of the underlying mathematics, yet now these shapes are more than abstract—they form the basis for the structure of spacetime. The frustums serve as discrete, quantized regions where physical laws begin to take hold, much like the quantization of spacetime hypothesized in quantum gravity theories.

4. Mathematical Laws Becoming Physical Laws:

    - The mathematical laws that govern the singularity—such as algebraic structures, geometric symmetries, and group theoretical rules—now begin to manifest as physical laws governing particles and forces. For example, symmetry principles like rotational invariance give rise to conservation laws such as the conservation of angular momentum.
    - The rules governing particle interactions, quantum fields, and spacetime curvature all emerge from the same underlying mathematics that defined the singularity. However, they now exist in a form that we recognize as physical reality. This transition is what allows quantum mechanical behaviors like tunneling and entanglement to arise in the frustums, which are the first pockets of spacetime created by this transition.

By framing the transition from mathematics to physics in this way, we suggest that the universe is fundamentally mathematical in origin. The physical laws we observe are simply the manifestation of deep, underlying mathematical rules that are present even in the singularity. The frustums represent the first regions where these mathematical rules transition into physical laws, providing a fascinating framework for exploring the birth of quantum mechanics, spacetime, and matter itself.

3. Frustums at the Planck Scale

Emergence of Frustums

As we move outward from the mathematical singularity, a crucial transition occurs at a critical radius where pure mathematical abstraction transforms into physical reality. This transition is marked by the emergence of frustums, geometric structures that represent the first manifestations of physical space. These frustums are not arbitrary but arise from the inherent symmetry and geometrical rules that exist in the singularity. As the first physical entities, frustums form the foundational building blocks of space at the Planck scale.

The frustums can be thought of as discrete regions of space, similar to the cells of a geodesic dome or the facets of a polyhedron. Each frustum originates at the critical radius from the singularity, forming a complex tiling pattern around the central point. The shapes of these frustums—whether pentagonal, hexagonal, or a combination of polygonal forms—are the first indicators of space's geometric structure. The key insight here is that space itself does not exist as a continuous fabric but is composed of these discrete geometric units that arise from the underlying mathematics of the singularity.

The edges of the frustums represent boundaries between different regions of space, and within each frustum, quantum mechanical phenomena such as particle confinement, tunneling, and interference begin to take place. The emergence of these frustums marks the moment when spatial regions—and hence spacetime—become well-defined, distinguishing themselves from the formless mathematical singularity that lies at the center.

Frustum Geometry and Planck Scale Physics

At the Planck scale, the geometry of the frustums is influenced directly by the singularity from which they originate. The shape of each frustum is governed by the symmetries and algebraic structures present in the singularity, which serve as blueprints for the formation of physical space. As these frustums emerge at the Planck scale, they encapsulate the quantum properties of the universe, providing the first stage where quantum fields and particles can be defined.

1. Discrete Geometric Units:
    - Each frustum can be thought of as a finite quantum region, akin to a pixel in a digital image but on a far more fundamental scale. These frustums tile the spherical surface at the Planck radius, creating a quantized version of space. Each one is distinct but connected to its neighbors through geometric relationships that originated in the singularity.
    - These frustums are not flat but curved, conforming to the spherical geometry of the space they occupy. The curvature of these frustums reflects the influence of gravity, which is still weakly defined at the Planck scale but becomes more prominent as the geometry unfolds into larger scales.
2. Quantum Mechanics in the Frustums:
    - Quantum fields and particles can be understood as existing within these frustums, with each frustum acting as a small quantum domain. Due to the extremely small scale of these regions, the quantum mechanical effects within them are profound. These frustums define the boundaries within which quantum mechanical wavefunctions are confined.
    - Inside each frustum, the behavior of particles is governed by the Schrödinger equation, modified for the curved geometry of the frustum. This confinement leads to quantum states that are influenced by the shape and size of the frustum, as well as the quantum interactions between neighboring frustums.
3. Frustums as Quantum Regions:
    - The frustum shape (whether pentagonal, hexagonal, or other geometric forms) plays a crucial role in determining the quantum properties of the particles within it. The discrete nature of space inside each frustum means that the quantum wavefunctions must conform to the boundaries imposed by the frustum geometry. These boundaries are not fixed; they can vary depending on the energy and quantum state of the particle, allowing for complex wavefunction

dynamics and quantum interference patterns across multiple frustums.

- Tunneling between frustums becomes a fundamental quantum process, as particles can quantum mechanically "leak" through the boundaries between frustums. This tunneling allows for the transfer of quantum states between neighboring regions, contributing to the emergent behavior of quantum fields at larger scales.

4. Frustum Curvature and Spacetime Emergence:
    - The curvature of the frustums reflects the emergent spacetime geometry. While the frustums at the Planck scale are small and highly quantized, they form the seeds for what will become the continuous spacetime that we observe at larger scales. The geometry of these frustums may also give rise to gravitational effects, as they represent the first physical manifestation of space curvature in the quantum regime.

Role of the Planck Scale

The Planck scale plays a central role in this model as the boundary between pure mathematical abstraction and the emergence of physical laws. The Planck scale is often cited in physics as the smallest length at which quantum mechanics, gravity, and spacetime are believed to be unified. It marks the point where classical descriptions of space and time break down, and quantum effects dominate. In our model, the Planck scale defines the critical radius from the singularity, where physical space first emerges in the form of frustums.

1. Quantum Gravity and the Planck Scale:
    - At scales larger than the Planck length, spacetime can be treated as a smooth, continuous fabric governed by general relativity. However, at the Planck scale, spacetime becomes discrete and quantum mechanical. This is the scale where quantum gravity effects are expected to become significant, where spacetime itself is thought to exhibit quantum properties such as granularity and superposition.

- The Planck scale is also the limit at which the uncertainty principle places a lower bound on the precision with which space and time can be measured. This uncertainty introduces fundamental quantum fluctuations in the geometry of spacetime, which manifest within the frustums as stochastic behavior of quantum fields.

2. Frustums as a Bridge Between Mathematics and Physics:
    - The frustums that emerge at the Planck scale serve as a bridge between the pure mathematics of the singularity and the physical reality of spacetime. Before reaching the Planck scale, reality is described purely by mathematical structures such as symmetries and geometric rules. After the Planck scale, these structures begin to take physical form, giving rise to the familiar laws of physics that govern matter, energy, and spacetime.
    - The Planck scale also represents the boundary where the quantum and classical worlds begin to diverge. At scales smaller than the Planck length, quantum effects dominate, and space becomes highly non-classical, while at larger scales, classical physics begins to emerge from these quantum foundations.
3. Why the Planck Scale is Fundamental:
    - The Planck scale is important because it is the smallest scale at which all known forces—including gravity—are believed to be unified. It represents the threshold where the quantum behavior of spacetime and matter becomes inseparable from the underlying mathematical structure.
    - At this scale, physical concepts such as mass, length, and time lose their classical meaning, and the universe operates according to quantum laws that are influenced directly by the underlying mathematics of the singularity. This makes the Planck scale a natural boundary between abstract mathematics and the emergence of physical space, time, and matter.

In summary, the frustums at the Planck scale represent the first manifestation of physical reality, arising from the pure mathematical singularity at the center. Their geometry is governed by the same mathematical symmetries that define the singularity, but within each frustum, quantum mechanical phenomena such as tunneling, entanglement, and interference begin to emerge. The Planck scale serves as the critical boundary where this transition occurs, marking the point where physical laws first come into existence from pure mathematics.

4. Quantum Mechanics in the Frustums

At the Planck scale, where the frustums first emerge from the mathematical singularity, quantum mechanical phenomena take center stage. These frustums, as discrete regions of space, encapsulate the first manifestations of quantum behavior, and each one is a quantum domain where particles can exhibit tunneling, superposition, entanglement, and interference. In this section, we explore how these fundamental quantum effects operate within and between the frustums.

Tunneling at the Planck Scale

Quantum tunneling is one of the first quantum phenomena to emerge in the frustums. Given the discrete nature of these frustums, they act like potential wells that confine particles within their boundaries. However, due to the quantum mechanical nature of particles, there is always a non-zero probability that a particle can "tunnel" through the boundary separating adjacent frustums.

1. Tunneling Between Frustums:
    - In classical physics, a particle would be confined to its frustum, unable to cross the potential barrier between adjacent frustums. In the quantum realm, however, particles can tunnel between these regions even though the potential barrier between them may be large.
    - Tunneling probability depends on the width of the boundary between frustums and the energy of the particle. Since the frustums are emergent at the Planck scale, where space itself is highly

quantized, these boundaries are expected to be thin, allowing for a significant tunneling effect between adjacent frustums. This tunneling could be the first observable quantum phenomenon, indicating that the boundaries between regions of space are not absolute, but permeable.

2. Wavefunction Overlap:
    - When particles tunnel between frustums, their wavefunctions overlap, creating regions of shared quantum information. This overlap could lead to interesting collective behaviors, where particles can move coherently across multiple frustums, creating quantum states that are shared across space.
    - Tunneling between frustums also suggests that space itself is not fixed, but a dynamic medium where quantum wavefunctions can spread across multiple regions. This is a reflection of how the underlying mathematical structures that define the frustums also allow for flexibility and interaction at the quantum level.
3. Emergence of Space Through Tunneling:
    - As particles tunnel between frustums, they essentially "connect" different regions of space. This suggests that tunneling could be one of the first quantum processes that leads to the emergence of physical space as a connected, continuous fabric. The interplay between tunneling particles could establish causal connections between different frustums, setting the stage for the larger-scale continuity of spacetime.

Superposition and Quantum States

Within each frustum, particles can exist in superposition, a hallmark of quantum mechanics. Unlike in classical physics, where particles occupy well-defined states, quantum particles can exist in multiple states simultaneously, each represented by a component of the wavefunction.

1. Superposition of Quantum States:
    - In the frustums, particles are not confined to a single quantum state but can exist in a superposition of states. This means that a particle in a frustum may simultaneously occupy multiple energy levels or quantum configurations, depending on the shape of the frustum and its boundary conditions.
    - At the Planck scale, the superposition can be especially significant because particles may be in a superposition of both mathematical and physical states. This reflects the dual nature of reality at the Planck scale, where space and time are still transitioning from mathematical abstractions into physical entities.
2. Superposition Across Frustums:
    - In addition to superposition within a single frustum, particles can also be in a superposition across multiple frustums. This is especially relevant when combined with tunneling, as particles can exist in a superposition of being confined to one frustum while simultaneously having a probability of existing in a neighboring frustum.
    - The quantum state of a particle may be spread over several frustums, creating a more complex superposition that reflects the underlying mathematical connections between regions of space. This superposition could serve as the basis for more complex quantum phenomena, such as quantum fields that extend across multiple frustums.
3. Interference of Superposition States:
    - Superpositions often give rise to quantum interference, where the different components of a particle's wavefunction interfere constructively or destructively. In the frustum model, superposition states could interfere both within a single frustum and between frustums, leading to complex interference patterns that reflect the underlying geometry of space.

- These interference patterns could provide a signature of the emergent physics in the frustums, as they would reveal how quantum states evolve as they transition from purely mathematical states at the singularity to physical states at the Planck scale.

Entanglement Across Frustums

Quantum entanglement occurs when two or more particles become correlated in such a way that the state of one particle instantly affects the state of another, regardless of the distance between them. In the frustum model, entanglement may reflect the deep, inherent mathematical connections between different regions of space that all originate from the same singularity.

1. Entanglement Between Frustums:
    - Because all frustums originate from the same singularity, there is a deep mathematical connection between them. This suggests that particles in different frustums could become entangled due to their shared origin, with their quantum states becoming correlated across vast distances.
    - Entanglement across frustums could indicate that the quantum structure of space itself is nonlocal, meaning that events in one frustum could instantly influence events in another, even though they are separated by a large distance in space. This reflects the fundamental interconnectedness of the underlying mathematical framework from which the frustums emerge.
2. Mathematical Origins of Entanglement:
    - Since the frustums originate from a singularity where only mathematics governs, the entanglement of particles may be a reflection of this deep mathematical symmetry. Entanglement could be seen as a natural consequence of the symmetries and geometric relationships that define the singularity, manifesting as correlated quantum states in the physical realm.

- This could provide a new way of thinking about quantum entanglement—not just as a physical phenomenon, but as a mathematical consequence of the universe's geometric structure.

3. Nonlocality and Spacetime Connectivity:
    - The existence of entanglement across frustums suggests that spacetime itself may be nonlocal at the quantum level. The quantum correlations between particles in different frustums could indicate that the separation between these regions is not absolute, but connected through a deeper, underlying mathematical framework. This could have profound implications for our understanding of spacetime and the nature of causality at the Planck scale.

Quantum Interference and Wavefunction Dynamics

As wavefunctions evolve from the singularity outward into the frustums, quantum interference patterns naturally arise. These interference patterns are an indicator of how physical phenomena emerge from the mathematical foundations of the singularity.

1. Interference Patterns in the Frustums:
    - As particles tunnel between frustums or exist in superposition across multiple frustums, their wavefunctions will interfere with each other. This interference can be constructive (where wavefunctions amplify each other) or destructive (where they cancel each other out), leading to complex interference patterns within the frustums.
    - These patterns provide insight into the dynamics of wavefunctions in the emerging quantum landscape, showing how particles interact with the boundaries of the frustums and how quantum states evolve over time. The geometry of the frustums will influence these patterns, as the wavefunctions must conform to the curved boundaries of the space.
2. Interference as a Signature of Emergence:
    - Quantum interference in the frustums is one of the first signs of physical laws emerging from the singularity. The interference

patterns reflect the interaction between quantum states and the geometry of space, indicating that physical space is beginning to organize itself according to the underlying mathematical structure.

- These patterns may also provide a way to study the transition from the abstract mathematics of the singularity to the physical laws that govern quantum fields and particles in spacetime.

3. Wavefunction Evolution:
    - As wavefunctions propagate outward from the singularity and interact with the frustums, they evolve in ways that reflect the underlying symmetries of the system. The evolution of these wavefunctions could be governed by a modified Schrödinger equation, taking into account the curvature and boundaries of the frustums.
    - This evolution may reveal how quantum fields themselves emerge from the mathematical framework, as the wavefunctions spread across multiple frustums and interfere with one another.

5. Mathematical Framework for Transition from Singularity to Physics

To describe the transition from pure mathematics at the singularity to the emergence of physical laws within the frustums at the Planck scale, we must develop a mathematical framework that accounts for how quantum mechanics and physical phenomena originate. This section presents a modified Schrödinger equation, defines the boundary conditions at the singularity, and establishes the Hamiltonian that governs the quantum behavior of particles as they transition from an abstract mathematical realm to a physical one.

Schrödinger Equation and Planck Scale Transition

The Schrödinger equation is a fundamental equation in quantum mechanics that describes how the wavefunction of a quantum system evolves over time. In the context of this model, where particles exist in discrete frustums emerging from a central singularity, the standard Schrödinger equation needs to be modified to

account for the unique geometry and boundary conditions imposed by the transition from pure mathematics to physics.

1. Modified Schrödinger Equation:
    - The traditional Schrödinger equation in flat, continuous space takes the form:

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$$

where ψ is the wavefunction, $\hbar$ is the reduced Planck constant, and $\hat{H}$ is the Hamiltonian operator representing the system's total energy. In our model, the equation must account for the **curved geometry** of the frustums, the influence of the **singularity**, and the **discrete nature** of the space inside each frustum.

    - To modify the Schrödinger equation for this context, we need to consider a curved spacetime metric that governs the geometry of each frustum. A general form of the modified equation in curved space is:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2_{\text{curved}}\psi + V_{\text{curved}}(r)\psi$$

where ∇^2_{curved} is the **Laplacian operator** in curved space (which accounts for the curvature of the frustums), and $V_{\text{curved}}(r)$ is the potential in curved space, which may vary as a function of the distance r from the singularity.

2. Wavefunction Behavior Near the Planck Scale:
    - As we approach the Planck scale, the nature of the wavefunction becomes heavily influenced by the underlying mathematical principles that govern the singularity. The wavefunction, ψ(r,t), near the Planck radius must reflect the discrete geometry of the frustums, where space itself is quantized into finite regions.
    - This quantization suggests that the wavefunction may have discrete energy levels within each frustum, similar to how particles in a potential well exhibit discrete quantum states. However, unlike the

infinite potential wells typically studied in quantum mechanics, the curved geometry of the frustums modifies the boundary conditions and introduces new constraints on the wavefunction.

3. Transition from Pure Mathematics to Quantum Physics:
    - The Schrödinger equation governs the transition from pure mathematics at the singularity to physical quantum mechanics in the frustums. Initially, near the singularity, the system is described purely by mathematical symmetries and abstract algebraic structures. As we move outward to the Planck scale, these symmetries begin to impose physical laws, and the Schrödinger equation captures this process by evolving the wavefunction from a state of pure symmetry to a quantum-mechanical one.
    - At the Planck scale, quantum fluctuations dominate, and the wavefunction becomes increasingly influenced by quantum mechanics, allowing for phenomena like tunneling, superposition, and entanglement to manifest within the frustums.

Boundary Conditions at the Singularity

The boundary conditions of the Schrödinger equation at the singularity are crucial to understanding how particles behave within the frustums. These boundary conditions determine how the wavefunction behaves at the central singularity and at the boundaries of the frustums, and they provide insight into how physical phenomena emerge from mathematical principles.

1. Boundary at the Singularity:
    - The singularity itself is a point where physical space does not yet exist. Therefore, the wavefunction at the singularity cannot be treated in the same way as it would be in a physical system. Instead, the wavefunction at the singularity must obey mathematical constraints derived from the symmetries and geometric properties of the singularity.
    - One possible boundary condition at the singularity is that the wavefunction must vanish at r=0 (the center of the singularity),

reflecting the fact that no physical particles can exist within the singularity itself. This would mean: $\psi(r = 0, t) = 0$. This boundary condition reflects the absence of physical space at the singularity and the purely mathematical nature of the central point.

2. Boundary at the Planck Scale:
    - At the Planck scale, the discrete frustums emerge as the first regions where physical space is defined. The wavefunction in each frustum must satisfy boundary conditions that reflect the geometry of the frustums. For example, the wavefunction may be constrained by the curved walls of each frustum, leading to quantized energy states within these regions.
    - A potential boundary condition for the wavefunction at the boundary of a frustum could be that the normal derivative of the wavefunction vanishes, indicating that there is no probability flux through the walls of the frustum:

$$\frac{\partial \psi}{\partial n}\Big|_{\text{boundary}} = 0$$

where $\frac{\partial \psi}{\partial n}$ represents the normal derivative of the wavefunction at the boundary of the frustum. This boundary condition implies that particles are confined within each frustum unless tunneling occurs.

3. Influence of Mathematical Symmetry:
    - The boundary conditions near the Planck scale are also influenced by the symmetries present at the singularity. For example, rotational symmetry around the central point may impose constraints on the angular components of the wavefunction, while other symmetries could dictate how the wavefunction behaves in relation to neighboring frustums.
    - These symmetries ensure that the transition from a purely mathematical system at the singularity to quantum mechanics in the

frustums follows consistent rules, with the geometry of the system reflected in the behavior of the wavefunction.

Hamiltonian for Emerging Physics

The Hamiltonian is the operator that represents the total energy of a system, including both the kinetic and potential energy of the particles. In the transition from the mathematical singularity to physical space at the Planck scale, the Hamiltonian plays a key role in governing how physical laws emerge from abstract principles.

1. Hamiltonian in Curved Space:
    - The curved geometry of the frustums must be accounted for in the Hamiltonian. In standard quantum mechanics, the Hamiltonian includes the kinetic energy term, which is usually expressed as:

$$\hat{H}_{\text{kinetic}} = -\frac{\hbar^2}{2m}\nabla^2$$

However, in our model, this term must be modified to reflect the **curved geometry** of the frustums. The **Laplacian operator** ∇^2 must be replaced with its curved-space counterpart ∇^2_{curved}, which accounts for the spherical and polygonal shapes of the frustums.

2. Potential Energy Term:
    - The potential energy term in the Hamiltonian, V(r)V(r)V(r), will depend on the distance from the singularity and the geometry of the frustums. Near the Planck scale, this potential may reflect the quantum effects that dominate at small scales, such as the uncertainty principle and quantum fluctuations. The potential energy in the frustums may also be influenced by the presence of neighboring frustums, creating a lattice-like potential that governs how particles interact across boundaries.

- One possible form of the potential energy term could be:

$$V(r) = V_0 \exp\left(-\frac{r}{L}\right)$$

where V_0 is a constant and L is the characteristic length scale (possibly on the order of the Planck length) that determines the strength of the potential at different distances from the singularity. This potential reflects the transition from a highly confined system near the singularity to a more spread-out system at larger radii.

3. **Interaction Hamiltonian:**

$$\hat{H}_{\text{tunnel}} = -t \sum_{\langle i,j \rangle} \left(\psi_i^\dagger \psi_j + \psi_j^\dagger \psi_i \right)$$

Here, t represents the tunneling amplitude, and the sum runs over neighboring frustums i and j. The operators $\psi_i^\dagger$ and $\psi_j^\dagger$ correspond to the creation of a particle in frustum i or j, while ψ_i and ψ_j represent the annihilation operators. This Hamiltonian describes how particles can tunnel between adjacent frustums, creating a quantum connection between the discrete regions of space.

6. Numerical Simulations and Visualization

Numerical simulations and visualization are key tools for investigating the behavior of quantum systems in the frustum model, where quantum mechanical phenomena such as tunneling, superposition, and interference play central roles. These methods will allow us to explore how quantum particles behave within the frustums at the Planck scale, how they interact with the boundary conditions imposed by the singularity, and how mathematical structures give rise to physical reality.

Simulating Quantum Tunneling and Interference

Simulating quantum tunneling and interference in the context of the frustums at the Planck scale requires a numerical approach that captures the curved geometry and discrete nature of the system. The following methods can be employed to numerically solve the modified Schrödinger equation, visualize wavefunction behavior, and simulate interactions between frustums.

1. Finite Element Method (FEM):
    - The Finite Element Method (FEM) is a powerful numerical technique for solving partial differential equations (PDEs), particularly in complex geometries. Given that the frustums are curved, non-Euclidean regions of space, FEM can discretize the system into smaller elements, allowing for the solution of the Schrödinger equation within each frustum.
    - FEM can handle the curvature and boundary conditions of the frustums, providing an accurate representation of how the wavefunction evolves within these regions. It can also simulate how particles tunnel between adjacent frustums by allowing for overlap of the wavefunctions at the boundaries.
    - To simulate tunneling, the FEM approach can model the interaction between neighboring frustums by adjusting the boundary conditions at their interfaces. The tunneling probability is calculated by allowing the wavefunction to penetrate the boundary, with the energy of the particle and the thickness of the boundary influencing the likelihood of tunneling.
2. Time-Dependent Schrödinger Equation (TDSE):
    - The time-dependent Schrödinger equation is crucial for simulating dynamic quantum systems. Numerical methods such as the Crank-Nicolson algorithm or Runge-Kutta integration can be used to solve the TDSE and track how the wavefunctions of particles evolve over time in the frustums.
    - Simulating quantum interference requires tracking the wavefunction as it spreads and interacts within and between frustums. As particles in superposition states tunnel across frustums, their wavefunctions can overlap, leading to constructive and destructive interference. By solving the TDSE, we can visualize how these interference patterns develop and evolve over time.
    - The initial conditions for the wavefunctions will depend on the geometry of the frustums and the boundary conditions imposed by

the singularity. The simulation can start with particles localized in individual frustums, and as they interact, the resulting interference patterns can be observed.

3. Spectral Methods:
    - Spectral methods are highly accurate for solving PDEs, especially when the problem involves periodic or smooth functions. These methods expand the wavefunction in terms of a set of basis functions (such as Fourier modes or spherical harmonics), which are well-suited to the symmetries of the frustums.
    - Spectral methods can be applied to simulate the wavefunction dynamics within the frustums by decomposing the Schrödinger equation into a set of coupled ordinary differential equations in the basis function space. This approach allows for efficient computation of wavefunction evolution, particularly in regions with high symmetry, such as the frustums near the singularity.
4. Lattice Models for Tunneling:
    - A lattice model approach can be employed to simulate the tunneling of particles between adjacent frustums. In this model, each frustum is treated as a lattice site, and the tunneling between frustums is represented by hopping terms in the Hamiltonian.
    - The tight-binding model, commonly used in condensed matter physics, can be adapted for this system. In the tight-binding approximation, the wavefunction is localized in each frustum, but particles can "hop" between neighboring frustums with a probability determined by the tunneling matrix element. This approach can simulate how particles explore the spatial regions of the frustums and how quantum interference arises as a result of tunneling.
    - The simulation can track the probability amplitudes of the wavefunctions at each frustum and compute the resulting interference patterns as the particles move across the lattice.

Visualizing the Transition from Symmetry to Physics

The transition from mathematical symmetry at the center (the singularity) to physical reality at the frustum boundary is a key aspect of the model. Visualization tools can help illustrate how quantum wavefunctions interact with the emerging physical structures and how the abstract mathematics of the singularity transforms into observable quantum phenomena.

1. Visualizing Wavefunctions in Curved Space:
    - One of the primary challenges in this model is visualizing the behavior of wavefunctions in the curved, discrete frustums. The wavefunction within each frustum can be visualized as a probability density function, showing the likelihood of finding a particle at a particular location inside the frustum.
    - To visualize how the wavefunctions evolve, we can use 3D plotting software or numerical visualization tools like MATLAB, Python (Matplotlib), or Paraview to create dynamic plots of the wavefunction at each time step. The wavefunction can be represented as a color-coded surface, where different colors correspond to different probability densities, allowing us to observe how the wavefunction interacts with the boundary of each frustum.
    - For interference patterns, visualization techniques can highlight regions of constructive and destructive interference by showing how the probability density oscillates as the wavefunctions interact with each other.
2. Mapping the Transition from Singularity to Planck Scale:
    - To visualize the transition from mathematical symmetry to physical space, we can construct a radial map starting from the singularity and extending outward to the Planck scale. The visualization can depict the increasing complexity of the wavefunction as it moves from the purely mathematical region near the singularity to the more structured, quantum-mechanical region of the frustums.

- This can be visualized as a series of concentric layers, with the innermost layer representing the singularity (where mathematical symmetry dominates) and the outer layers representing the frustums at the Planck scale (where physical laws begin to emerge). Each layer can show the evolving wavefunction and how it interacts with the boundary conditions at each radius.

3. Frustum Geometries and Boundary Conditions:
    - The geometric structure of the frustums can also be visualized to highlight how the wavefunctions interact with the boundary conditions imposed by the curved walls of each frustum. Using software like Blender or Unity, we can create 3D models of the frustums and show how particles are confined within them.
    - The visualization can demonstrate how wavefunctions behave at the boundaries of the frustums, either reflecting off the walls or tunneling through to neighboring frustums. This can be particularly useful for visualizing how tunneling events occur and how wavefunctions spread across multiple frustums.
4. Visualization of Symmetry Breaking:
    - As the system transitions from the mathematical singularity to physical space, the symmetry breaking that occurs at the Planck scale can be visualized. Symmetry breaking refers to the process where the perfect mathematical symmetries near the singularity give way to the more complex, less symmetric structures of physical space.
    - This can be visualized by showing how the geometric symmetries of the frustums (such as their pentagonal or hexagonal shapes) emerge from the highly symmetric mathematical structure of the singularity. The breaking of symmetry can be represented as a gradual change in the structure of the frustums as they grow larger and more complex at the Planck scale.

5. Dynamic Evolution of Tunneling and Superposition:
    - A key aspect of the model is the dynamic evolution of quantum states, particularly how particles in superposition tunnel between frustums. Dynamic visualizations can show how the wavefunction changes over time, highlighting regions where tunneling occurs and where superposition leads to interference effects.
    - Animations of the wavefunction evolution can illustrate how quantum states evolve as particles tunnel from one frustum to another, and how their wavefunctions interfere as they propagate through the system.
6. Visualizing Entanglement:
    - Quantum entanglement can be visualized by showing how particles in different frustums become correlated with one another. This can be done by illustrating the correlations between the wavefunctions of entangled particles, highlighting how the measurement of one particle affects the state of another.
    - Visualization tools can display entanglement links between frustums, with lines or color gradients indicating the degree of entanglement between particles in different regions. This can help to show the nonlocal nature of quantum mechanics and how entanglement spans across multiple frustums.

7. Physical Interpretation and Cosmological Implications

The model outlined in this paper, where a central singularity governed by pure mathematical principles transitions into physical reality through the emergence of frustums at the Planck scale, offers profound implications for our understanding of the early universe and the nature of cosmological evolution. In this section, we will explore how this framework could provide new insights into cosmological theories, particularly concerning the origin of the universe, the emergence of physical laws, and potential experimental systems that might model this transition from mathematics to physics.

Cosmological Interpretation

In the context of cosmology, the model can be viewed as a theoretical framework for understanding the origins of the universe, specifically the transition from a pre-physical, mathematically governed singularity (analogous to the Big Bang singularity) to the emergence of space, time, matter, and physical laws. This interpretation draws on the notion that before the Planck scale, the universe existed purely as a set of mathematical constructs, with no physical meaning for concepts such as time or space. It is only as we move outward from this central singularity, reaching the Planck scale, that physics as we understand it begins to emerge.

1. The Singularity as Pure Mathematics:
    - In standard cosmological models, the Big Bang singularity is often treated as a point where the known laws of physics break down, and quantities like temperature and density approach infinity. However, in this model, the singularity is interpreted as a point of pure mathematical abstraction, where symmetries, geometric relationships, and algebraic structures exist without physical manifestation.
    - The singularity could represent a state where spacetime itself has no definition, and only abstract mathematical rules apply. In this interpretation, time, space, and matter do not yet exist in any meaningful sense—reality is described entirely by mathematical principles. The transition from this state to the physical universe occurs as the system crosses the Planck scale.
2. Emergence of Physical Laws at the Planck Scale:
    - As we move outward from the singularity, reaching the Planck scale, the physical universe begins to emerge. In this model, the Planck scale acts as a critical boundary where quantum fluctuations of spacetime and matter begin to dominate, and the abstract mathematical symmetries of the singularity give rise to quantized regions of space, represented by the frustums.

- At this scale, physical laws such as the conservation of energy, the behavior of quantum particles, and the structure of spacetime begin to take form. This implies that physical reality—including the laws of physics we observe—emerges from mathematical structures and is shaped by the discrete geometry of the frustums. The geometry of the universe at its earliest moments is therefore encoded in the structure of these frustums, which expand and evolve as the universe grows.

3. Implications for the Big Bang:
    - This model suggests that the Big Bang itself is not simply a physical event, but rather the birth of physical laws from a pre-physical, mathematically governed state. The initial conditions of the universe, which have long puzzled cosmologists, may be understood as the first manifestation of physical geometry through the frustums.
    - The Planck scale, in this interpretation, is not just the limit where current physical theories break down, but the actual threshold where the mathematical becomes physical, allowing for the formation of spacetime and matter.

New Models of the Early Universe

This framework opens the door to new models of the early universe, particularly those that explore the nature of the universe before the emergence of physical laws at the Planck scale. This model posits that space and time themselves do not exist in any meaningful way before the Planck scale, and that the transition from pure mathematics to physics provides new ways to think about the origin of the universe.

1. Pre-Physical Universe:
    - One of the key insights of this model is that before the universe reaches the Planck scale, there are no physical laws in place—no spacetime, no matter, and no forces. This implies that the origin of the universe could be understood not as the creation of matter and energy, but as the emergence of space and time from underlying

mathematical structures. The singularity is purely abstract, governed by principles that are not yet physically manifested.

- As we move closer to the Planck scale, quantum gravity becomes important, and spacetime itself becomes discrete, represented by the frustums. This discrete structure could explain why space and time appear continuous on large scales but are inherently quantized at smaller scales.

2. Quantum Geometry and Early Universe Dynamics:
    - The discrete frustums could provide a model for quantum geometry, a concept in quantum gravity where spacetime itself has a grainy structure at very small scales. These frustums, as discrete regions, could be viewed as the first elements of this quantum geometry, providing a way to understand the early dynamics of the universe.
    - This model suggests that the universe's initial expansion from the singularity is not a smooth process, but rather a quantum-driven expansion where the structure of spacetime is built out of frustums, expanding outward and forming the initial conditions for the universe's subsequent growth.
3. Implications for Cosmological Inflation:
    - Cosmological inflation, the rapid expansion of the universe in its earliest moments, could be reinterpreted in this model. Instead of being a purely physical expansion of spacetime, inflation could represent the rapid unfolding of mathematical structures into physical laws, with the frustums forming and expanding rapidly as physical space emerges from the singularity.
    - This could also offer insights into how quantum fluctuations during inflation become the seeds for the large-scale structure of the universe, as these fluctuations could be linked to the interference patterns and quantum tunneling between frustums.

Experimental Analogues

While the model described here is highly theoretical, aspects of it may be explored through experimental analogues, particularly in fields such as quantum computing, high-energy physics, and quantum gravity simulations. By developing experiments that mimic the transition from mathematical rules to physical phenomena, we can potentially test some of the concepts proposed by this model.

1. Quantum Computing Experiments:
    - Quantum computers provide a natural platform for exploring the transition from mathematical rules to physical phenomena, as quantum computation itself operates on purely abstract principles that govern quantum states. Experiments could be designed to simulate the discrete quantum states within the frustums, allowing us to model how wavefunctions behave under the influence of curved geometry and quantum tunneling between frustums.
    - In particular, quantum simulation algorithms could be used to mimic the dynamics of the frustums and their boundaries, providing insight into how quantum mechanical properties like superposition, entanglement, and interference manifest in a system governed by discrete regions of space.
2. High-Energy Physics Simulations:
    - High-energy physics experiments, such as those conducted at particle accelerators like the Large Hadron Collider (LHC), could offer insight into the behavior of particles in extreme conditions, potentially probing scales near the Planck length. By studying the results of particle collisions at incredibly high energies, we may be able to infer how quantum fields behave in highly curved, discrete regions of space, much like the frustums in this model.
    - Numerical simulations in quantum gravity, using tools like lattice quantum field theory or loop quantum gravity, could be used to explore how spacetime quantization affects the behavior of particles

and fields at small scales, providing a testbed for ideas related to the discrete geometry of the frustums.

3. Quantum Gravity Theories:
    - This model shares similarities with some approaches in quantum gravity, such as loop quantum gravity and causal set theory, which posit that spacetime is fundamentally discrete. Experimental efforts in these areas, such as attempting to detect quantum fluctuations of spacetime or the discreteness of spacetime structure, could provide indirect evidence for the ideas proposed in this model.
    - Gravitational wave detectors could also offer insights into the geometry of spacetime at very small scales. If the discrete frustum structure leaves an imprint on gravitational waves, it might be possible to detect subtle signatures of spacetime quantization in the data from instruments like LIGO or Virgo.

8. Conclusion

This paper has presented a novel framework for understanding the transition from pure mathematics to physical reality, centered around the concept of a mathematical singularity at the core of existence. This singularity represents a pre-physical state where only mathematical laws and symmetries exist, with no manifestation of space, time, or matter. As we move outward from this singularity, we reach a critical radius, possibly at or near the Planck scale, where physical laws begin to emerge. At this boundary, the discrete regions known as frustums are formed, marking the first appearance of quantized space, time, and matter.

Summary of the Novel Concept

At the heart of this model is the idea that mathematics precedes physics. The singularity, governed by abstract principles like geometric symmetry, group theory, and algebraic relationships, represents the foundation from which physical laws eventually arise. The transition from this purely mathematical state to the physical world occurs as the system moves outward toward the Planck

scale. Here, quantum mechanics takes over, providing the foundation for the emergence of space, time, and matter.

This transition is represented by the formation of frustums, discrete regions of space where physical phenomena begin to manifest. The frustums, which emerge from the central singularity, represent a natural tiling of space, governed by the symmetries and structures present at the singularity. Within these regions, quantum mechanics plays a pivotal role, driving phenomena such as:

- Quantum tunneling: Particles confined within the frustums are subject to the quantum mechanical process of tunneling, where they can penetrate the boundaries between frustums. This tunneling behavior reflects the permeability of space at small scales, highlighting how quantum mechanics governs interactions across these boundaries.
- Superposition: Particles in this framework can exist in a superposition of states, both within individual frustums and across multiple frustums. The wavefunction, which represents the quantum state of the system, evolves within the curved geometry of the frustums, reflecting the influence of the singularity and the quantum nature of space itself.
- Entanglement: The frustums, while spatially distinct, are inherently connected through the deeper mathematical symmetries that define the singularity. Particles in different frustums may become entangled, such that the quantum state of one particle is linked to the state of another, regardless of the physical distance between them. This entanglement reflects the nonlocality of quantum mechanics and the profound mathematical connections underlying the universe.

Unique Role of Quantum Mechanics in the Transition

Quantum mechanics serves as the bridge between the mathematical singularity and the emergence of physical reality. It is at the Planck scale, within the frustums, that the core principles of quantum mechanics—such as wave-particle duality, superposition, and entanglement—come into full effect. These quantum phenomena are the first signs of physical law, suggesting that the universe's physical structure is fundamentally quantum mechanical, arising from deep mathematical principles.

In this framework:

- Tunneling suggests that the boundaries between different regions of space are not absolute but permeable, reflecting the inherent uncertainty and flexibility of quantum mechanics.
- Superposition implies that particles can exist in multiple states simultaneously, reflecting the probabilistic nature of reality at small scales.
- Entanglement reveals that space is not truly separable, even across distances, as particles maintain deep, nonlocal connections due to the mathematical structure from which they emerge.

Future Directions

This model opens up several potential avenues for future exploration, particularly in the fields of quantum gravity, cosmology, and theoretical physics. The following are key directions that could further refine and test the ideas presented in this paper:

1. Refining the Model with More Detailed Simulations:
    - Future research could focus on performing more detailed numerical simulations of the quantum behavior within the frustums. These simulations could explore how the wavefunctions evolve over time, how quantum tunneling occurs between frustums, and how interference patterns develop as particles interact.
    - Simulations of the Schrödinger equation in curved spaces could provide more precise predictions for how quantum mechanics governs the emergence of physical reality in this model, particularly at the Planck scale. Incorporating quantum gravity effects into these simulations could offer new insights into how gravity, spacetime, and quantum mechanics interact in the early universe.
2. Exploring Cosmological Implications:
    - This model has profound implications for our understanding of cosmology, particularly regarding the origin of the universe. Future work could explore how this framework relates to the standard

cosmological models, such as inflationary theory or the Big Bang singularity.

- Research could also investigate how the discrete frustum structure influences the evolution of the early universe. Does this structure provide a natural explanation for the quantization of spacetime in quantum gravity theories? Could this model offer a new perspective on cosmological inflation, where space expands as a result of frustums unfolding from the singularity?

3. Implications for Quantum Gravity and Theories of Everything:
    - The transition from mathematics to physics described in this paper could provide a useful framework for developing a Theory of Everything—a unified theory that combines quantum mechanics, general relativity, and fundamental mathematical structures. By exploring how physical laws emerge from mathematical principles, this model could offer a new pathway toward understanding the quantization of spacetime in the context of quantum gravity.
    - Future research could focus on how this framework fits into existing theories of quantum gravity, such as loop quantum gravity or string theory. How does the discrete nature of the frustums compare to the quantized loops or vibrating strings proposed in these models? Can this framework help resolve the contradictions between general relativity and quantum mechanics?
4. Experimental Investigations:
    - While much of this model is highly theoretical, future work could focus on designing experimental analogues that simulate the transition from mathematics to physics. Quantum computing experiments, high-energy physics simulations, or even experiments in gravitational wave detection could provide indirect evidence for the ideas presented in this paper.
    - Research could also focus on detecting potential signatures of quantum spacetime discreteness through observations of gravitational waves or cosmic microwave background (CMB)

fluctuations. If spacetime has a discrete structure at small scales, it may leave observable imprints in high-energy astrophysical phenomena.

Final Thoughts

In conclusion, this paper has proposed a new framework for understanding the origin of physical laws, suggesting that pure mathematics lies at the heart of reality. As we move outward from a central mathematical singularity, physical laws begin to emerge, with quantum mechanics playing a central role in this transition. This model offers new insights into the early universe, the nature of spacetime, and the quantum structure of reality, laying the groundwork for future theoretical and experimental explorations that could deepen our understanding of the universe's most fundamental principles.

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